Brightness Illusions as Optimal Percepts

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Abstract

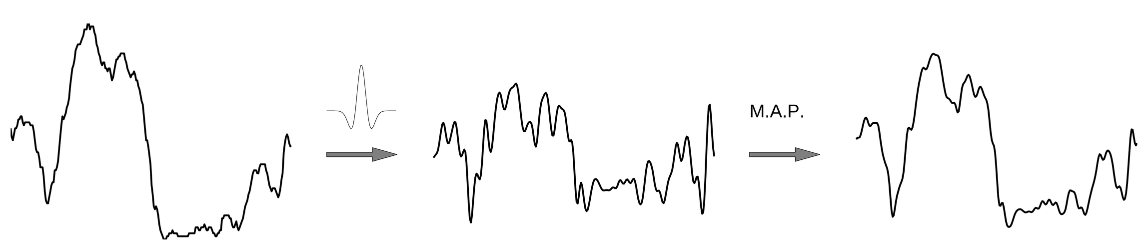
We show that several brightness illusions can be largely accounted for by assuming that the perceptual system performs simple Bayesian inference using a Gaussian image prior with noisy retinal ganglion cells. This theory accounts for phenomena which have been problematic for lateral-interaction models while avoiding the complexities of mid-level vision theories that involve the estimation of structure and albedo.

Models for brightness illusions

- Lateral inhibition (Ratliff, 1965).
- Color constancy models (Adelson, 2000).
- Physiological models (Albright, 1995).
- *Optimal perception*.

Optimal inference and the brain

The study of motion perception has revealed a rich set of seemingly counterintuitive non-veridical percepts. These motion illusions are well explained by a recent theory which posits that the visual system performs optimal perceptual inference in estimating motion, and that this processing necessarily takes into account both signal and noise (Weiss et al., 2002). Here we introduce a philosophically similar theory, extending the optimal perception hypothesis to encompass low-level brightness illusions without invoking complex priors that involve three-dimensional structure, grouping, illumination, transparency, or albedo.



Transformation and inference process. The stimulus (left) is convolved with the retinal transformation function (*Mexican hat*) to produced a new signal (middle). The brain has to "guess" the original pattern (here using a *maximum a-posteriori* estimation approach) from a noisy version of the transformed signal.

Methods

The input to the model presented here is a one-dimensional discrete signal (corresponding, for example, to one row of an image) which represents the lightness of the stimulus. The intensity range of the pixels is centered at zero (representing mid-level gray). Negative values correspond to darker tones, and positive values to lighter tones.

The transformation of the stimulus $\mathbf{x}$ at our one-dimensional retina is modeled as

$$\mathbf{y} = \mathbf{\Phi}\,\mathbf{x} + \eta$$

where $\mathbf{\Phi}$ is a circulant matrix and η is the noise due to transduction and transmission. Each row of $\mathbf{\Phi}$ corresponds to a shifted version of a *Mexican hat*-shaped function defined by

$$\phi(k) = a(b - k^2)e^{-k^2/2}$$

where a determines the power of the signal and b defines the area ratio between the positive and negative lobes. This transformation thus corresponds to convolving the input with a *Mexican hat* function. The noise η is assumed Gaussian with covariance $\mathbf{C}_\eta$, zero-mean, and uncorrelated with the input signal.

Given $\mathbf{y}$ as the signal received from the retina, we assume that the brain knows how to estimate the stimulus $\mathbf{x}$ that maximizes the posterior probability

$$p(\mathbf{x}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$$

The $\mathbf{x}$ which maximizes the consequent log-likelihood

$$\log p(\mathbf{x}|\mathbf{y}) = -\frac{1}{2\sigma_\eta^2}(\mathbf{y}-\mathbf{\Phi x})^T\mathbf{C}_\eta^{-1}(\mathbf{y}-\mathbf{\Phi x}) - \frac{1}{2\sigma_\mathbf{x}^2}\mathbf{x}^T\mathbf{C}_\mathbf{x}^{-1}\mathbf{x} - \mathrm{k}$$

must satisfy the linear equation

$$(\mathbf{\Phi}^T\mathbf{C}_\eta^{-1}\mathbf{\Phi} + \mathbf{C}_\mathbf{x}^{-1})\mathbf{x} = \mathbf{\Phi}^T\mathbf{C}_\eta^{-1}\mathbf{y}$$

If we assume a noise covariance $\mathbf{C}_\eta = \sigma_\eta^2\mathbf{I}$, and a white Gaussian prior with covariance $\mathbf{C}_\mathbf{x} = \sigma_\mathbf{x}^2\mathbf{I}$, this becomes

$$(\sigma_\eta^{-2}\mathbf{\Phi}^T\mathbf{\Phi} + \sigma_\mathbf{x}^{-2}\mathbf{I})\mathbf{x} = \sigma_\eta^{-2}\mathbf{\Phi}^T\mathbf{y}$$

which is equivalent to the Wiener filter for the deconvolution of a noisy signal.

References

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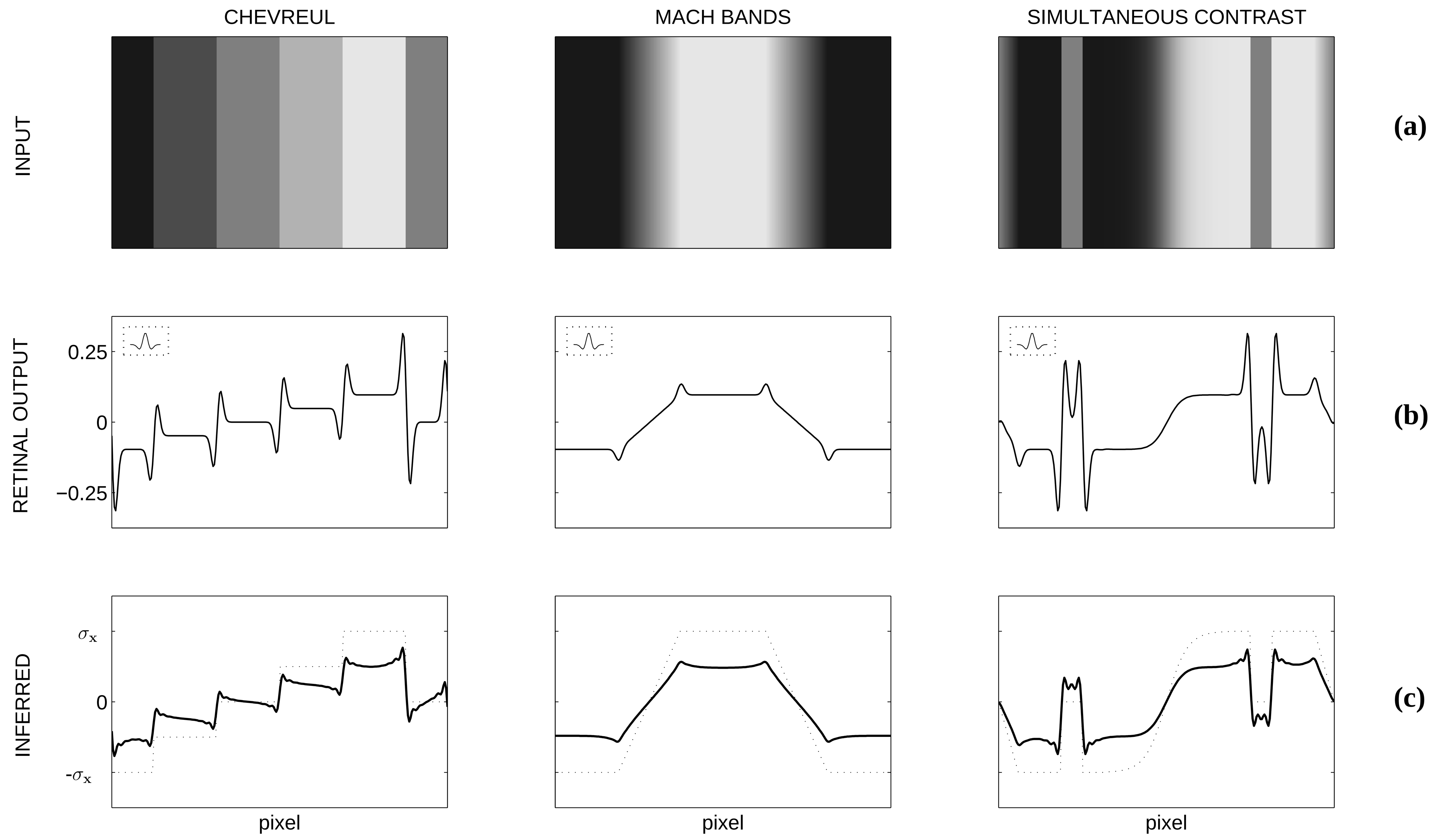
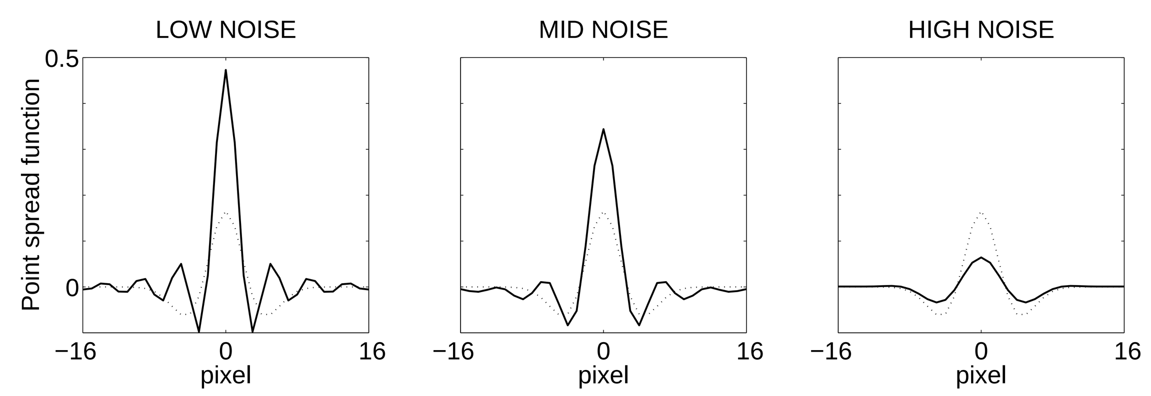
Weiss, Y., Simoncelli, E. P., and Adelson, E. H. (2002). Motion illusions as optimal percepts. *Nature Neuroscience*, 5(6):598–604.



Results

Optimal perceptual inference for three patterns. **(a)** Stimuli as images. **(b)** Transformed pattern after convolution with the *Mexican hat* function. The width of the Mexican hat is shown on the top-left corner of each plot. **(c)** Inferred pattern (solid) for each stimulus (dotted).

The stimuli consist of 256 pixels, with the maximum lightness set to $\sigma_\mathbf{x}$. The effective support (non-zero region) of the Mexican hat signal used for these simulations is approximately one tenth of the length of the input pattern.



Point spread function. The retinal processing impulse response is shown (dotted line) along with the impulse response function of the composition of the retinal processing and the optimal reconstructor (solid line). In all three panels $\sigma_\mathbf{x} = 1$, while $\sigma_\eta = 0.01, 0.1, 1.0$ (moving from left to right).

The results are consistent with reported percepts for several illusory patterns. In the Chevreul/Staircase pattern the bars with homogeneous lightness are perceived as ramps. Mach Bands (peaks of brightness) appear at the edges between a ramp and a plateau. And, for the simultaneous contrast stimulus, the gray bar on the light background seems to be darker than the gray bar on the dark background, even though the two gray bars have the same actual intensity.

Conclusions

As in other sensory domains, a general Bayesian principle and a simple generic prior has proven surprisingly powerful at accounting for low-level brightness illusions.

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